Semester One Examination, 2017

Question/Answer booklet

MATHEMATICS SPECIALIST UNIT 3

Section Two: Calculator-assumed

If required by your examination administrator, please place your student identification label in this box

Student Number: In figures | | | | | | | | |

In words _____

Your name _____

Time allowed for this section

Reading time before commencing work: ten minutes
Working time: one hundred minutes

Materials required/recommended for this section

To be provided by the supervisor
This Question/Answer booklet
Formula sheet (retained from Section One)

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: drawing instruments, templates, notes on two unfolded sheets of A4 paper, and up to three calculators approved for use in this examination

Important note to candidates

No other items may be taken into the examination room. It is **your** responsibility to ensure that you do not have any unauthorised material. If you have any unauthorised material with you, hand it to the supervisor **before** reading any further.

Structure of this paper

Section	Number of questions available	Number of questions to be answered	Working time (minutes)	Marks available	Percentage of examination
Section One: Calculator-free	8	8	50	52	35
Section Two: Calculator-assumed	12	12	100	98	65
				Total	100

Instructions to candidates

1. The rules for the conduct of examinations are detailed in the school handbook. Sitting this examination implies that you agree to abide by these rules.

2. Write your answers in this Question/Answer booklet.

3. You must be careful to confine your response to the specific question asked and to follow any instructions that are specified to a particular question.

4. Additional working space pages at the end of this Question/Answer booklet are for planning or continuing an answer. If you use these pages, indicate at the original answer, the page number it is planned/continued on and write the question number being planned/continued on the additional working space page.

5. Show all your working clearly. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat any question, ensure that you cancel the answer you do not wish to have marked.

6. It is recommended that you do not use pencil, except in diagrams.

7. The Formula sheet is not to be handed in with your Question/Answer booklet.

See next page

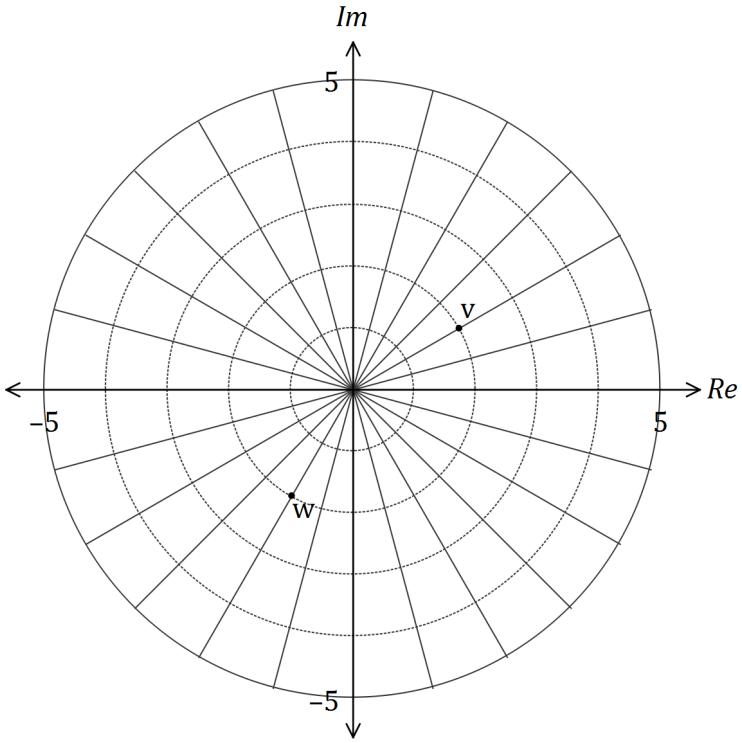
Section Two: Calculator-assumed **65% (98 Marks)**

This section has **twelve (12)** questions. Answer **all** questions. Write your answers in the spaces provided.

Working time: 100 minutes.

Question 9 **(6 marks)**

The complex numbers v and w are shown on the Argand diagram below.



On the diagram, clearly mark the complex numbers

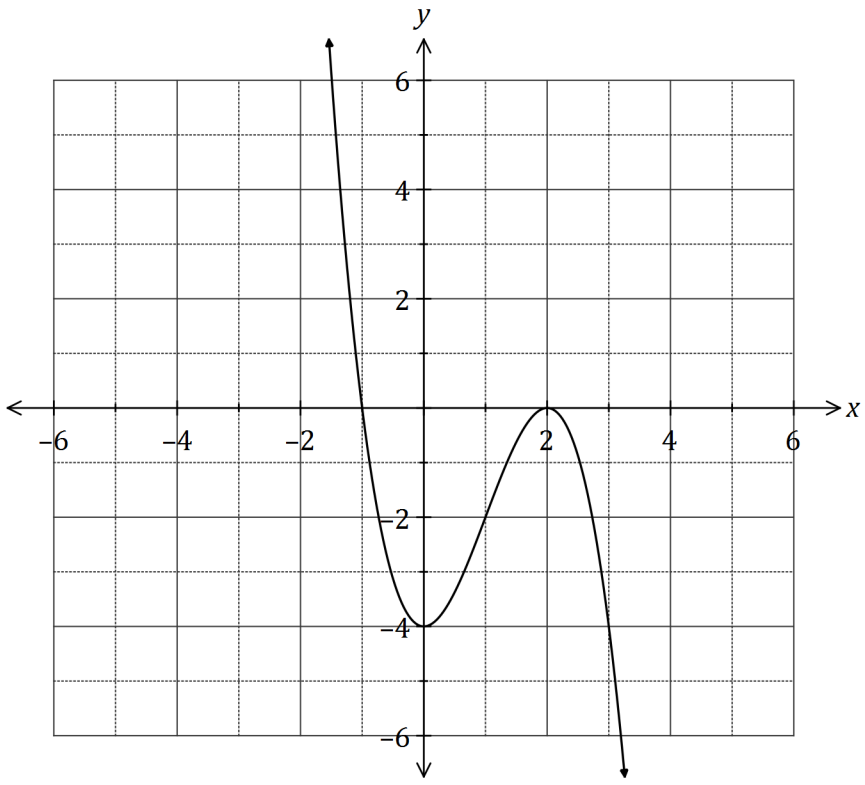
(a) $z_1 = vw$. (2 marks)

(b) $z_2 = \frac{v}{w}$. (2 marks)

(c) $z_3 = v - iw$. (2 marks)

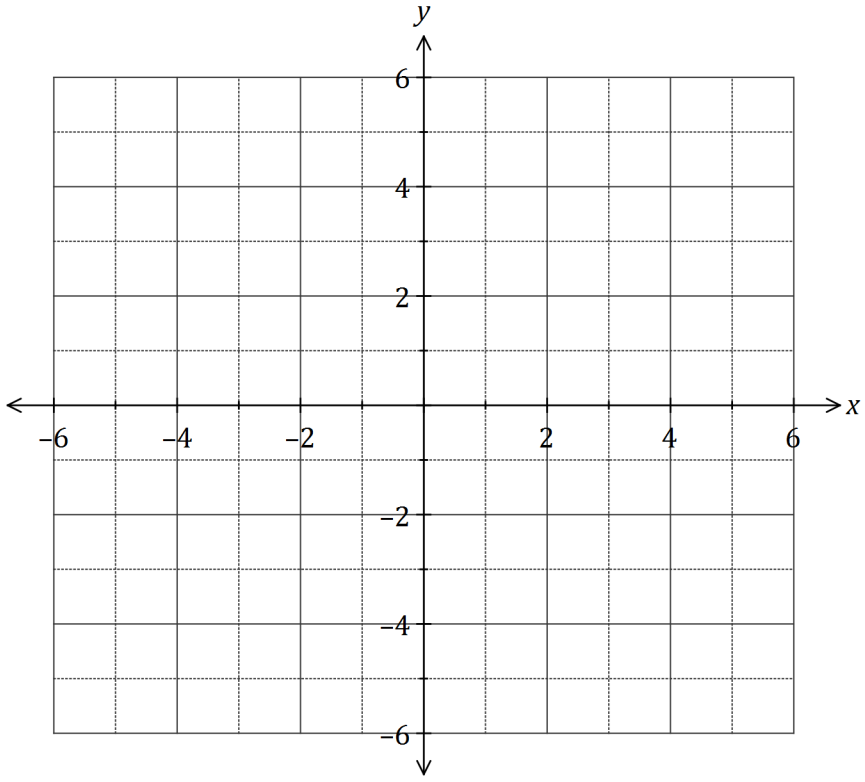
Question 10 **(8 marks)**

The graph of $y = f(x)$ is drawn below.



On the axes provided, sketch the graphs of

(a) $y = f(|x|)$. (2 marks)



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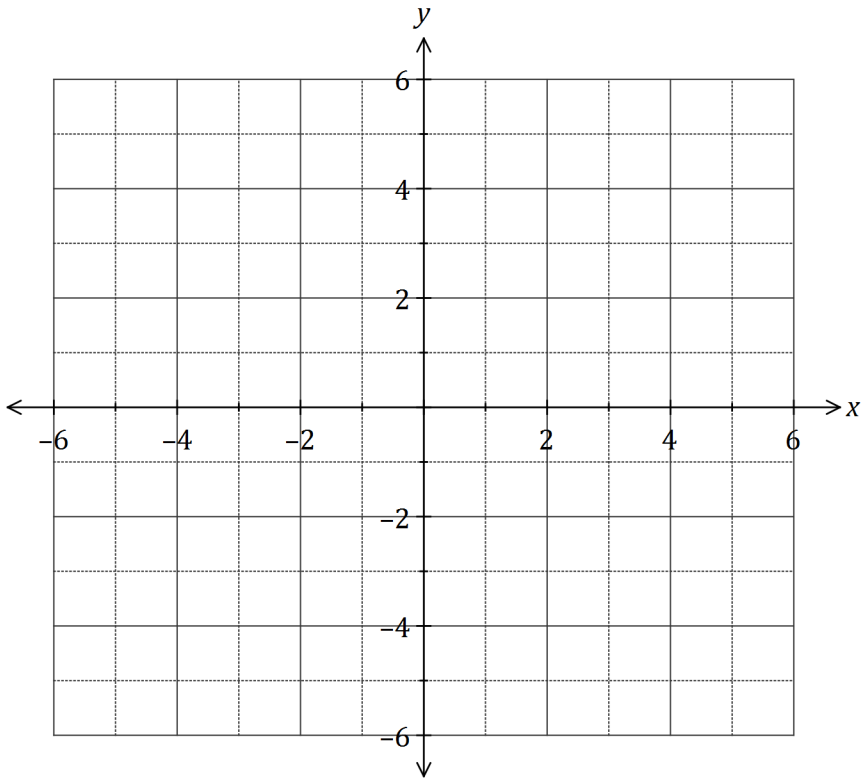
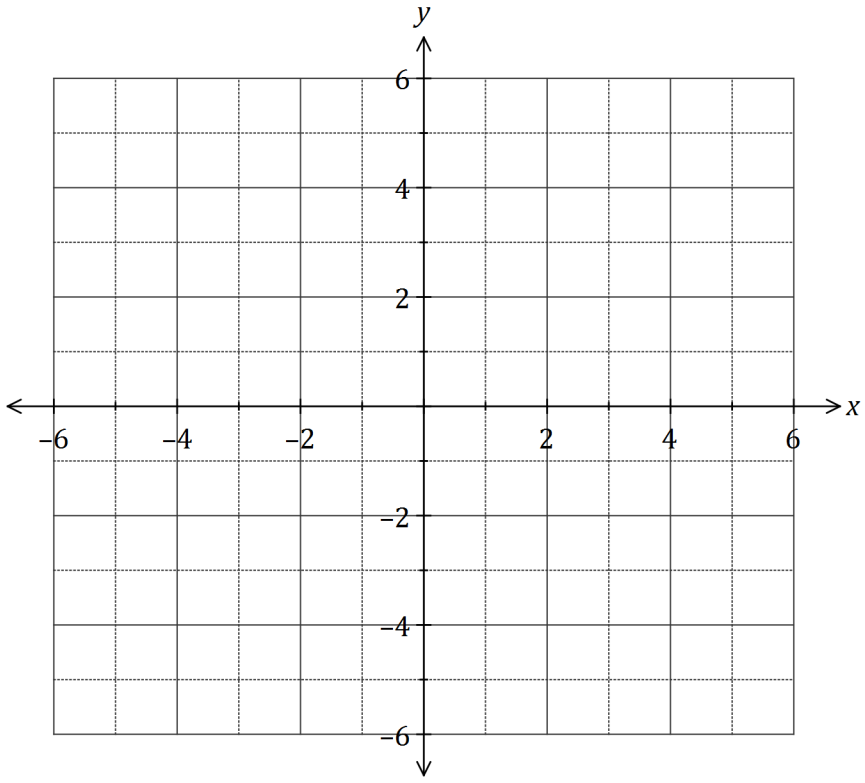
(b) $y = f^{-1}(x)$, the inverse relation. (2 marks)



(c) $y = \frac{1}{f(x)}$. (4 marks)



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Question 11 **(6 marks)**

The position vector $\boldsymbol{r}(t)$ of a model railway train at time t, in an appropriately chosen coordinate system, is given by

$$\boldsymbol{r}(t) = 3\cos kt\ \boldsymbol{i} - 2\sin kt\ \boldsymbol{j}$$

where distances are measured in metres and time is measured in seconds after an appropriately chosen starting time. The number k is positive.

(a) Obtain a Cartesian equation for the path traversed by the train. (3 marks)

(b) Describe the geometric shape of the path of the train. (1 mark)

(c) Does the train travel in a clockwise or anticlockwise direction around its closed path? Justify your answer. (2 marks)

See next page

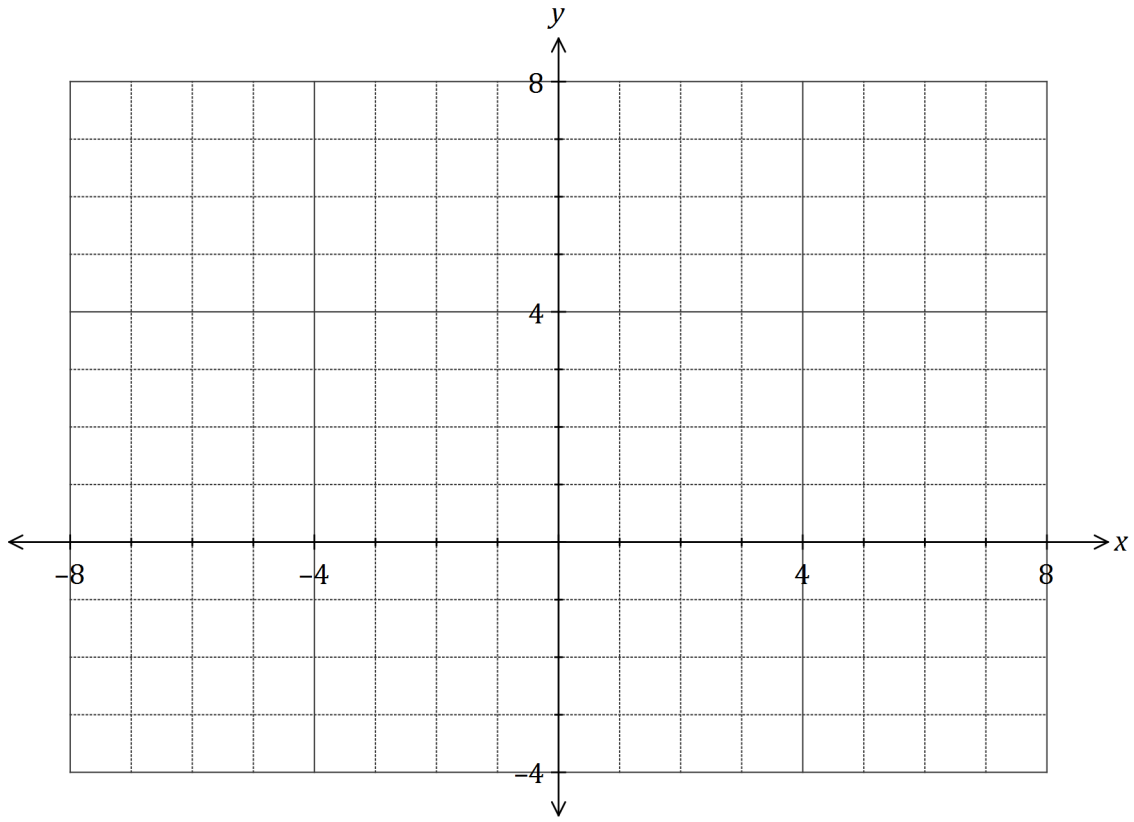
Question 12 **(9 marks)**

A function is defined by $f(x) = \frac{x^2+4x-12}{3x-7}$, $x \neq 0$.

(a) Determine the exact coordinates of all stationary points of the graph of $y = f(x)$. (2 marks)

(b) Determine the equation(s) of the asymptote(s) of the graph $y = f(x)$. (3 marks)

(c) Sketch the graph $y = f(x)$ on the axes below. (4 marks)



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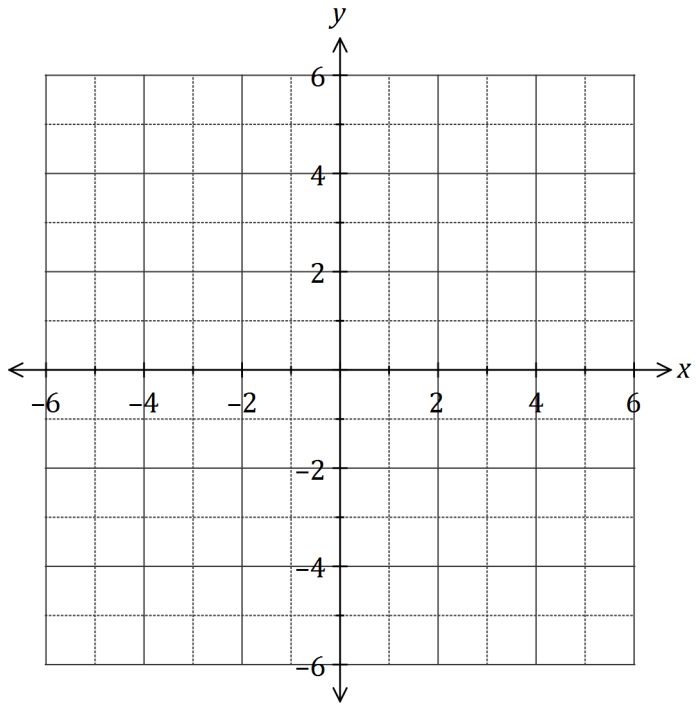
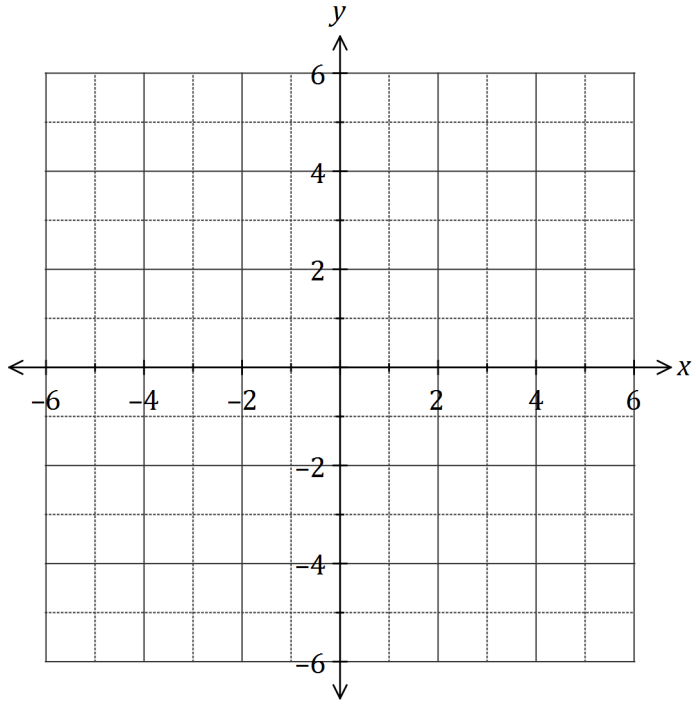
Question 13 **(11 marks)**

(a) On the Argand planes below, sketch the subsets of the complex plane determined by

(i) $|z + 3i| = |z + 2 - i|$. (3 marks)



(ii) $|z + 3 + i| \leq 3$. (3 marks)



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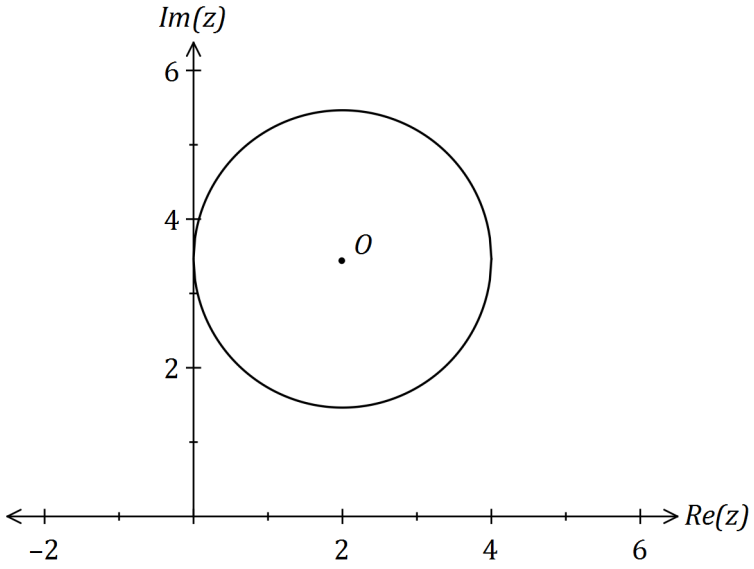
(b) A subset of the complex plane, a circle with centre O, is shown below.



(i) Mark the position in the plane where $|z|$ is maximised. Label this point (i).
(1 mark)

(ii) Mark the position in the plane where $|z-2|$ is minimised. Label this point (ii).
(1 mark)

(iii) If the subset shown is $\left|z-2-2\sqrt{3}i\right|=2$, determine the maximum and minimum values of $\arg z$. (3 marks)

Question 14 **(8 marks)**

The plane P has equation $\mathbf{r} \cdot \mathbf{n} = 11$, where $\mathbf{n} = \mathbf{i} - \mathbf{j} + 2\mathbf{k}$ and the point A has position vector $2\boldsymbol{i} + 5\mathbf{j} - 2\mathbf{k}$.

(a) Determine the Cartesian equation of plane Q that is parallel to P and passes through A. (2 marks)

(b) Determine the equation of the line L that passes through A and is perpendicular to P. (1 mark)

(c) Determine the position vector of B, the point of intersection of line L with plane P. (3 marks)

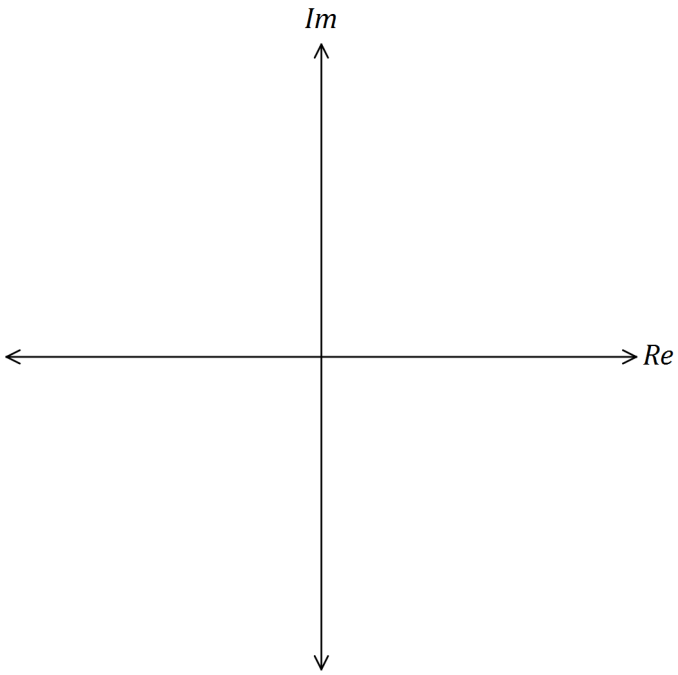
(d) Determine the exact distance between planes P and Q. (2 marks)

Question 15 **(8 marks)**

Consider the complex equation $z^5 = -16 + 16\sqrt{3}i$.

(a) Solve the equation, giving all solutions in the form $r \operatorname{cis} \theta$, $r > 0$, $-\pi \leq \theta \leq \pi$. (4 marks)

(b) Plot the solutions found in part (a) on the Argand diagram below, indicating all key features of the plot. (4 marks)



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Question 16 **(10 marks)**

The plane P intersects the axes at the points $A(-3,0,0)$, $B(0,4,0)$ and $C(0,0,1)$.

(a) Demonstrate use of the cross product to find vector $\boldsymbol{n}$ that is normal to the plane P. (2 marks)

(b) Find a Cartesian equation for P. (2 marks)

A vector equation for the line L is $\boldsymbol{r} = (2 + 3\lambda)\boldsymbol{i} + 4\boldsymbol{j} + (1 + \lambda)\boldsymbol{k}$.

(c) Demonstrate the use of dot product to show that line L n does not intersect plane P. (3 marks)

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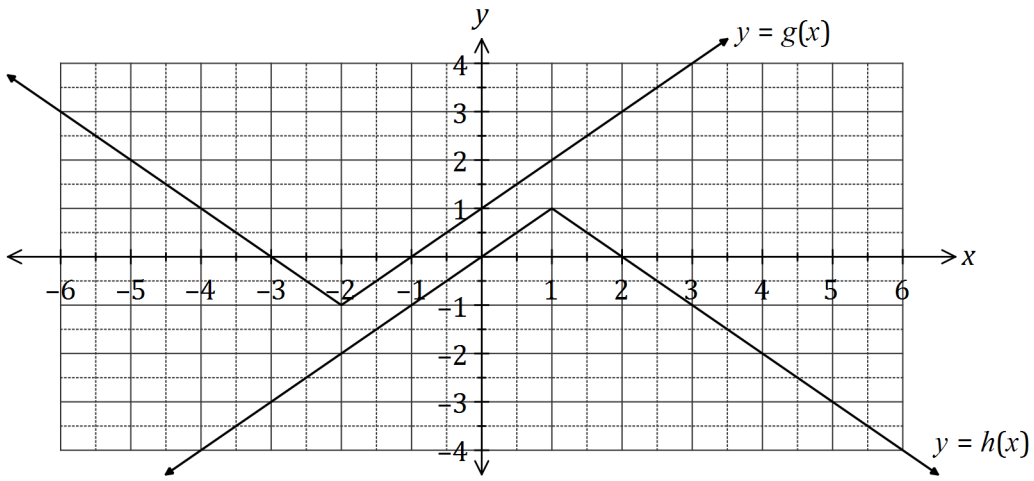
(d) Determine the distance between plane P and line L. (3 marks)

Question 17 **(9 marks)**

(a) The graphs of the functions g and h are shown below.

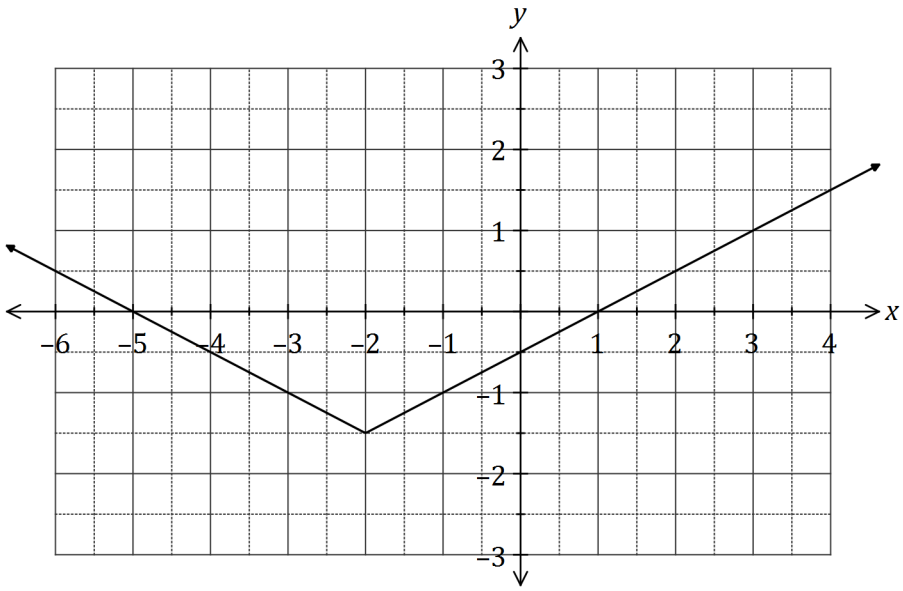


Determine the value(s) of k if

(i) $k = h \circ g(2)$. (1 mark)

(ii) $g\big(h(k)\big) = 1$. (2 marks)

(b) The graph of $f(x) = a|x - p| + q$ is shown below.



(i) Determine the value of the constants a, p and q. (3 marks)

(ii) If the equation $|f(x)| = mx + c$ has an infinite number of solutions, determine the values of the positive constants m and c. (3 marks)

See next page

Question 18 **(10 marks)**

The Cartesian equation of the sphere S is

$$x^2 - 6x + y^2 + z^2 + 10z = 2.$$

(a) Determine the radius and the coordinates of the centre of the sphere S. (3 marks)

The vector equation of the line L is

$$\boldsymbol{r}(t) = (9 + 2t)\boldsymbol{i} - 2t\boldsymbol{j} + (1 + t)\boldsymbol{k}$$

(b) Does the line L intersect the sphere S and if so, where? (5 marks)

(c) Explain why L is tangential to S. (2 marks)

See next page

Question 19 **(6 marks)**

Determine, where possible, a unique solution for the following systems of equations. In each case, interpret the system of equations geometrically.

(a) $8x + y + z = 15$, $2x + y - z = 3$, and $x - y + 2z = 3$. (2 marks)

(b) $x + y - z = 0$, $x - y + 2z = 10$ and $3x - y + z = 16$. (2 marks)

(c) $x + y = z + 2$, $x - y + z = 1$ and $x + z = y + 3$. (2 marks)

See next page

Question 20 **(7 marks)**

Given the following system of equations for variables x, y, z.

$$\begin{cases} 2x - 3y + z = 4 \\ x - y + \boldsymbol{p}z = 3 \\ x - 2y + 2z = \boldsymbol{p^2} \end{cases}$$

(a) Use Gaussian elimination to show that the given system of equations always has at least one solution.

(5 marks)

(b) Describe clearly the conditions required on $\boldsymbol{p}$ so that the system of equations has only one solution. (2 marks)

End of questions

Additional working space

Question number: _____